In which content to specialize? A game theoretic analysis

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Abstract. Content providers (CPs) may be faced with the question of how to choose in what content to specialize. We consider several CPs that are faced with a similar problem and study the impact of their decisions on each other using a game theoretic approach. As the number of content providers in a group specializing in a particular content increases, the revenue per content provider in the group decreases. The function that relates the number of CPs in a group to the revenue of each member may vary from one content to another. We show that the problem of selecting the content type is equivalent to a congestion game. This implies that (i) an equilibrium exists within pure policies, (ii) the game has a potential so that any local optimum of the potential function is an equilibrium of the original problem. The game is thus reduced to an optimization problem. (iii) Sequences of optimal responses of players converge to within finitely many steps to an equilibrium. We finally extend this problem to that of user specific costs in which case a potential need not exist any more. Using results from crowding games, we provide conditions for which sequences of best responses still converge to a pure equilibrium within finitely many steps.

1 Introduction

We consider in this paper competition over content type between Content Providers (CPs). We consider the sitation in which each of several CPs has to decide in what content to specialize. Different types of content may differ by their popularity as well as by the price that idividuals are willing to pay. We may expect that not all CPs will specialize in the most popular content since the income for other content may be larger if its distribution is shared among a small number of CPs.

We first show that this game is equivalent to a congestion game and thus has a potential. Any local maximizer of the potential is thus an equilibrium, and equilibrium can be reached by best response sequences within a finite number of moves [4, 2].

We extend some of these results to player specific costs in which case there need not be any potential. still a pure equilibrium is seen to exist and we establish convergence to it within finitely many steps under some conditions.

2 The model

There are M content providers and K content types. We consider below a game in which each content provider has to decide in what content type it specializes. We assume that it cannont specialize in more than one type.

Define a multi-policy $u = (u_1, ..., u_M)$ where u_i is the content type chosen by CP i, taking an integer value between 1 to K.

Let $\gamma^i(u)$ be the utility of content provider i ($i = 1, ..., M$) when the multi-policy u is used.

Let $\mathbf{m} = \{m_1, ..., m_K\}$ be the vector of the number of CPs specializing in each one of the K content types. We call this the system's state. We shall denote by $\mathbf{m}(u)$ the state that correspond to a multi-policy u.

We shall assume that the utility of CP i for choosing action k depends on the system state $\mathbf{m}$ only through its kth component m_k. i.e. it is a function of only how many CPs (including i) choose content k. We denote this utility as $J^i(k, m_k)$.

Let u be a multi-policy such that $u_i = k$ for some i and k and let $m_k = m_k(u)$. Then

$$J^i(k, m_k(u)) = \gamma^i(u).$$

In the next section we shall assume that $J^i(k, m_k)$ does not depend on i (it ia then omitted from the notation). This is a game with symmetric utilities (or costs). This assumption will be relaxed in the following section.

We consider the set satisfying

$$\mathbf{G}(M) := \{\mathbf{m} : m_i \geq 0,\ i = 1, ..., M,\ m_1 + ... + m_K = M\}$$

Define $S(\mathbf{m}) = \{i : m_i\ 0\}$ to be the support of $\mathbf{m}$, and let e_j denote the unit vector of dimension M with all entries zero except the jth that is one. This set thus contains all possible states in which each of the M CPs has chosen one content type between 1 and K..

Definition. $\mathbf{m}^*$ is said to be an equilibrium in the content game if

$$J(k, m_k^*) \geq J(i, m_i^* + 1), \quad \forall k \in S(\mathbf{m}) \text{ and all } i \neq k.$$

3 Analysis of the game

Theorem 1. *The following hold:*

– *(i) There exists a pure equilibrium in the content game.*

– *(ii) Define the following potential:*

$$V(\mathbf{m}) = \sum_{k=1}^{K} v(k, m_k), \ \ \textit{where } v(k,j) = \sum_{i=1}^{j} J(k,i)$$

Consider the problem of maximizing the potential V, i.e.

$$Z := \max_{\mathbf{m} \in \mathbf{G}(M)} V(\mathbf{m})$$

Let Q be the subset of $\mathbf{G}(M)$ achieving the max. Then any $\mathbf{m} \in Q$ is an equilibrium in the content game.

Proof. The content game is equivalent to a congestion game as defined in [3], where there is one common source and destination, there are M players and K parallel links and each player has one unit of flow to ship, and has to decide over which link to send it (it is not possible to split the flow between several links). The cost for a player to choose link k if there are m players that choose this link (including itself) is $J(k, m)$. With this equivalence, all statements follow from [3]. $\diamond$

Assume next that the following assmption holds:

A1: $J(k, i)$ is decreasing in i for all k.

Under this assumption the game is not only a congestion game but also a crowding game. As such, we know [2] that that best responses of players converge **within finitely many steps** to an equilibrium (provided that players do not change their strategies simultaneously, and that each player has an opportunity to update its strategy as long as an equilibrium is not reached). This convergence property is called the "Finite Improvement Property" [2].

We next discuss the practical motivaations for Assumption A1 throgh an example. Assume that each CP can satisfy a demand of L downloads per day. Let $D(k, p)$ be the Demand (in downloads per day) for type k content, provided that the price for downloading a unit of such content is p. D is assumed to be strictly decreasing in the price p. Therefore the following inverse function

$$P(k, d) = \{p : D(k, p) = d\}$$

is well defined. It represents the cost of type k content for which the resulting demand is of d downloads per days.

If there are m CPs of type k then the available offer for type k content is mL downloads per day, so that the price of a download that will create a demand that will match this offer is $P(k, mL)$.

The income of a type k CP when there are m CPs specializing in that content is then given by

$$J(k, m) := LP(k, mL).$$

4 Extensions

We study an extension to games with non-symmetric costs and then briefly suggest an extension to the elastic case.

4.1 Player specific costs

In [1] we have studied game problems involving two types of content providers: one that corresponds to independent content providers, and one that correspond to content providers that have exclusive agreements with Internet Service Providers (ISPs). The cost for the Internauts who are subscribers of some ISP of fetching content from an independent CP or from ma CP that has an exclusive agreement with another ISP, was assumed to be larger than for fetching it from the a CP that has an exclusive agreement with their own ISP. This impilies that the revenues (and thus the utility) of a CP may depend on its type (independent or not).

This motivates us to allow the utility function that corresponds to choosing some content to be player specific. We thus abandon the assumption of indistinguishability. The game is no more a congestion game and need not have a potential any more. The utility $J^i(k, m_k)$ may depend now on i.

Still the following holds:

Theorem 2. *Assume A1. The following holds in the case of player dependent costs:*

- *(i) There exists a pure equilibrium in the content game.*
- *(ii) In case that there are only two types of contents, the finite improvement property still holds.*

The proof follows Theorems 1 and 2 of [2].

4.2 Other extension

Both the results of the previous Section as well as the previous subsection have been derived for the case where all CPs participate. We next show how to apply them to the elastic case, i.e. where the offer for content may be a function of the utility that a CP receives. We wish to model a situation in which some CPs may prefer not to participate, if their income goes below some threshold.

We vary the model as follows. We add a new action "0" to each player, which corresponds to having an option of not participating in the game. In addition, we consider a fixed additional cost $c_i \geq 0$ for player i to participate in the game (which may correspond to the investment and maintainance costs). This problem can again be solved with the help of the equivalent congestion game, in which we add an additional parallel link of a fixed cost 0, and where we add c to the cost of each other link. Thus the same results as obtained previously still hold.

5 Conclusions

We have shown how various problems related to competition over content can be reduced to congestion games and/or to crowding games. This allowed us to derive the structure of equilibria and convergence to equilibria within finitely many improvement steps.

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